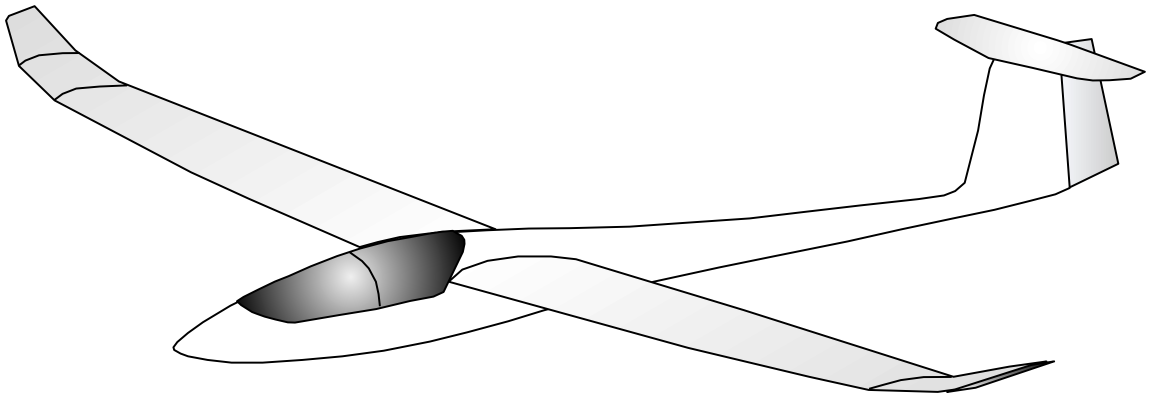


Triangle-based Galerkin panel methods

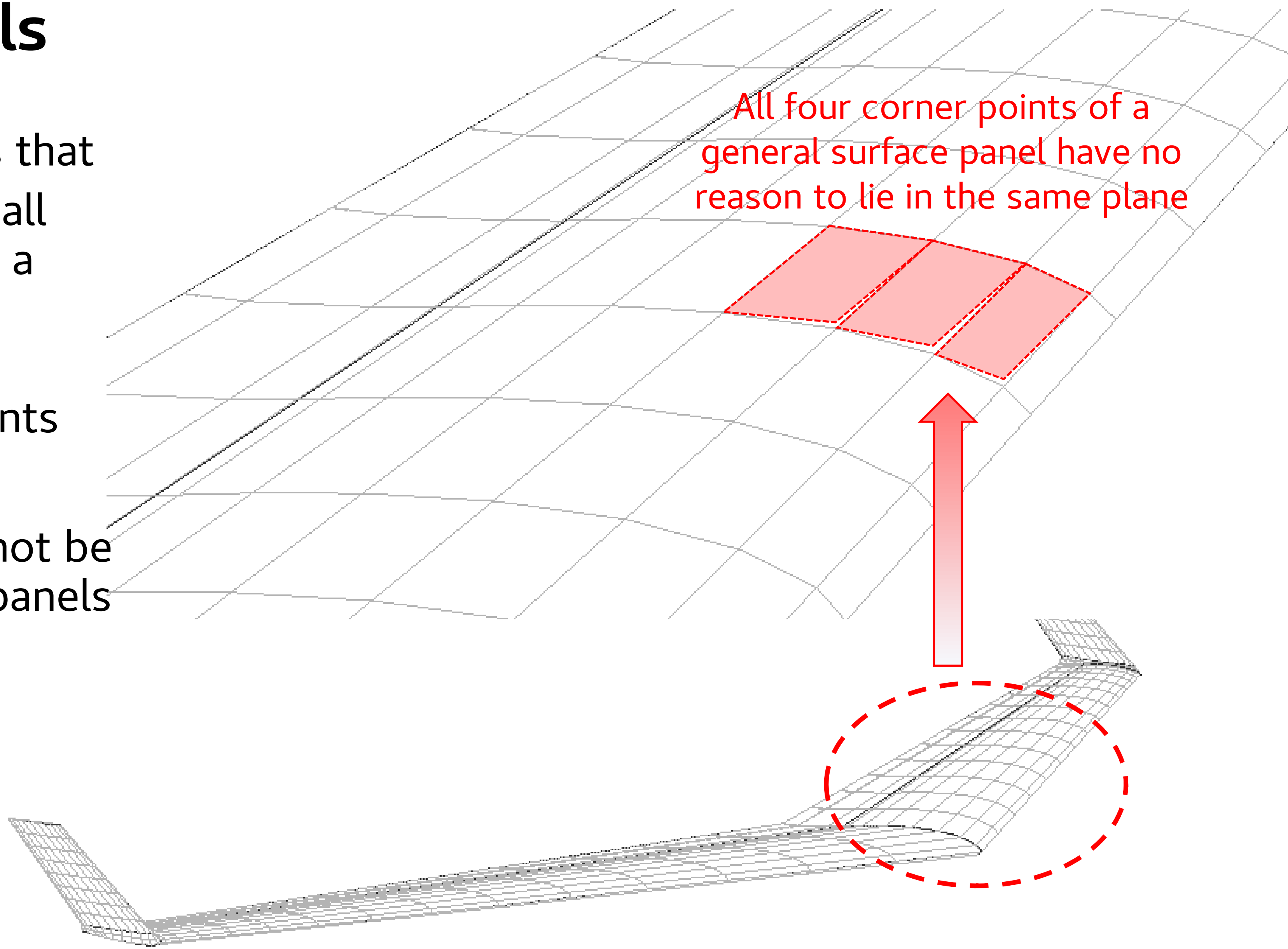
Motivation

- Lift the two restrictions associated to xflr5’s quad panels method
  (1) Impossibility to cover general 3d surfaces with flat quad panels
  (2) Limited to uniform source and doublet densities

- A triangle-based method is necessary to model fuselage-wings connections.

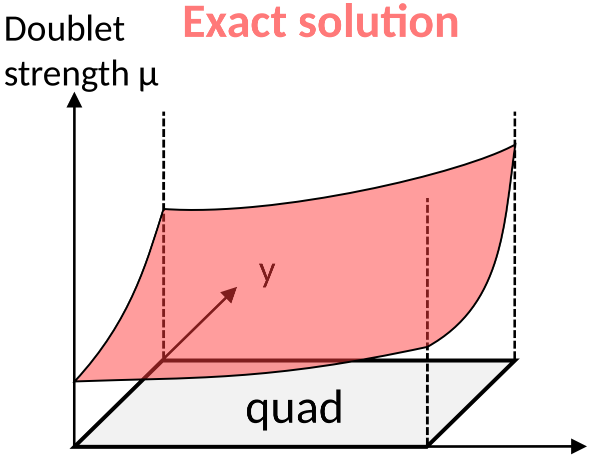
(1) Flat quad panels

- The 3d panel method requires that
  (1) quad panels are flat, i.e. all four corner points are in a plane
  (2) the volumes are entirely closed by surface elements
- However
  - 3d surfaces generally cannot be decomposed in flat quad panels



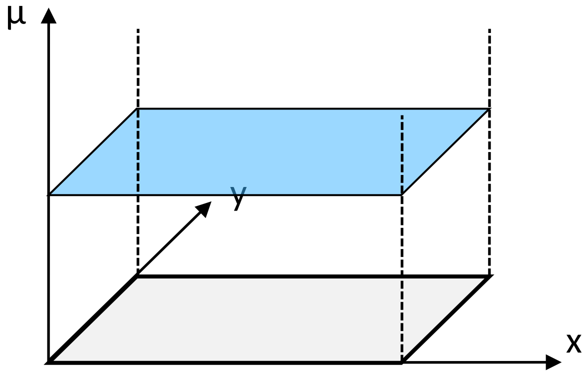
(2) Uniform strength singularities

- Panels method in xflr5 are of the <u>uniform type</u>
  - Uniform vortex strength on each panel in the case of the VLM
  - Uniform source and doublet strengths in the case of the panel method
- Methods of higher order are more precise



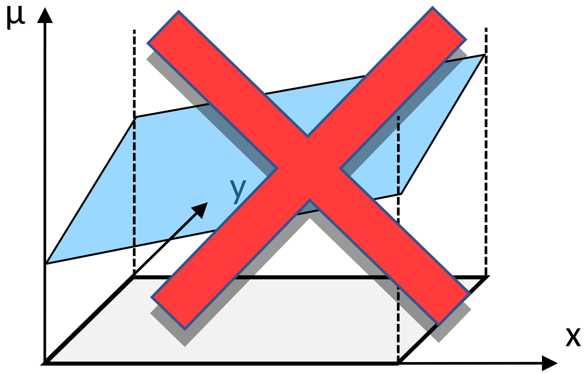
$\mu = \mu(x, y)$

Uniform model order 0



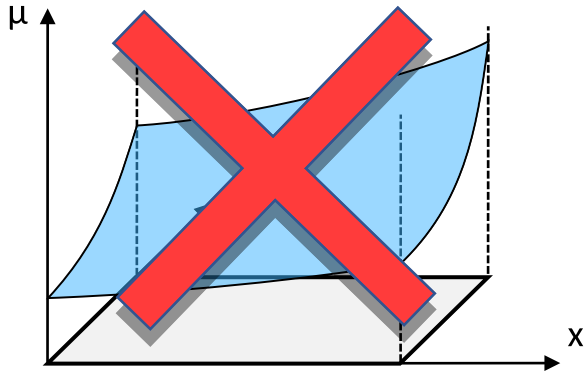
$\mu = \mu_c$

Linear model order 1



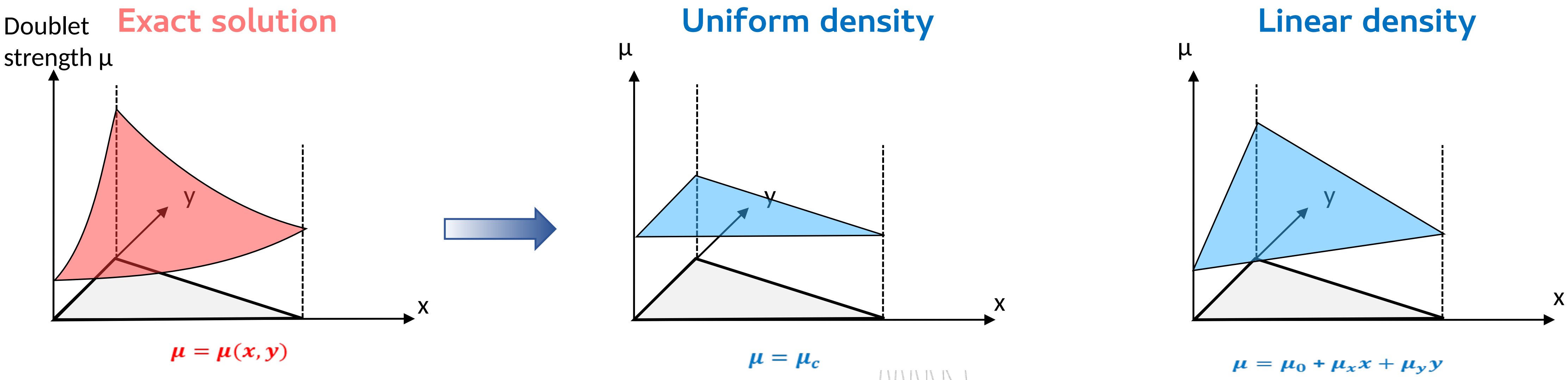
$\mu = \mu_0 + \mu_x x + \mu_y y$

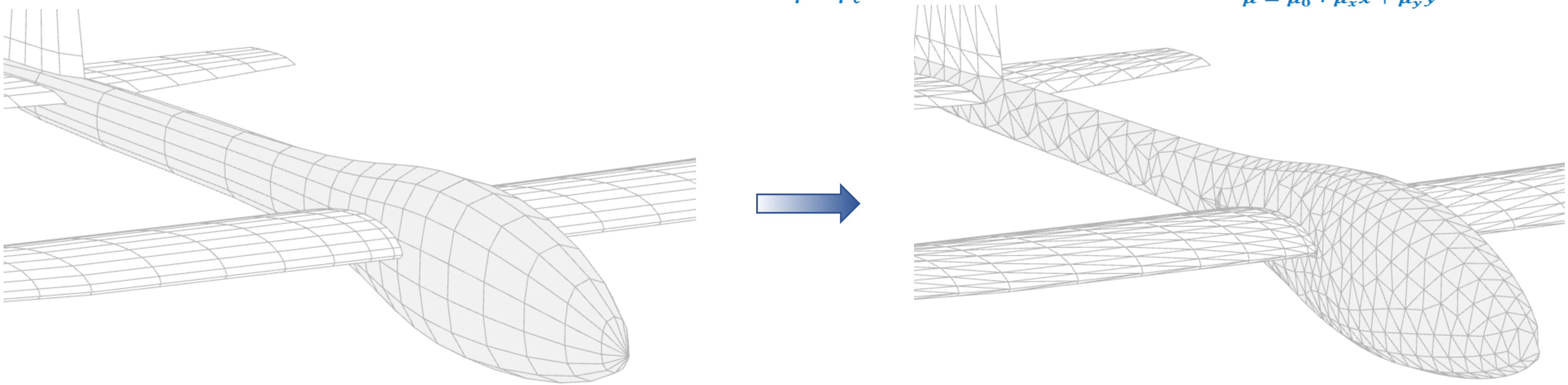
Quadratic model order 2



$\mu = \mu_0 + \mu_x x + \mu_y y + \mu_{xy} xy + \mu_{x2} x^2 + \mu_{y2} y^2$

A triangular Galerkin method solves all three issues





About the triangular methods in flow5

1. flow5 replaces each quad panel with two triangles → Twice as many elements

2. The linear method uses 3 degrees of freedom (d.o.f.) for each triangle instead of 1 d.o.f. for the uniform method
    - The linear system for the linear method is 3x the size that of the uniform method
    - The LU decomposition time of the system's matrix increases as 2/3 n^3

→ Not an issue with the super-powerful Intel® MKL library

About the triangular methods in flow5 (cont.)

3. Matrices used in Boundary Element Methods (BEM) are dense and not sparse like in F.E. analysis
   ⇒ memory footprint increases as n^2

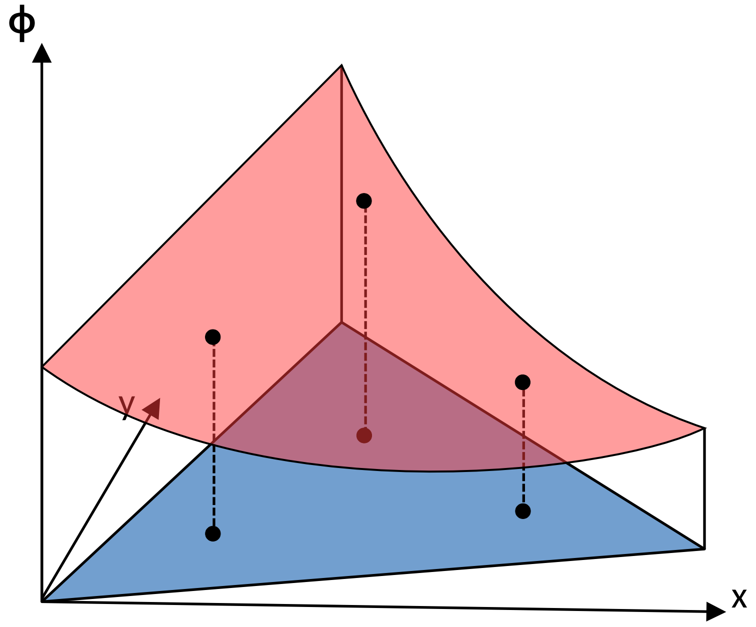
   Matrix sizes up to 15k x 15k are possible on a 8GB PC

4. The linear method requires the evaluation of surface integrals.

   Matrix and RHS contruction times are longer than for uniform methods

About the triangular methods in flow5 (cont.)

- The linear method involves the calculation of surface integrals on the triangles
- These integrals are performed using Gaussian quadrature
- The greater the number of quadrature points, the better the precision.
- The times to construct the influence matrix and the RHS vectors will increase proportionally to n_q



Order 2 → 3 quadrature points

Order 1:	1 quadrature point
Order 2:	3 quadrature points
Order 3:	4 quadrature points
Order 4:	6 quadrature points
Order 5:	7 quadrature points
Order 6:	12 quadrature points
Order 7:	13 quadrature points
Order 8:	16 quadrature points

Recommendation: $3 \leq$ order ≤ 5

About the triangular methods in flow5 (cont.)

5. Galerkin methods can be used with mixed Dirichlet and Neumann BC

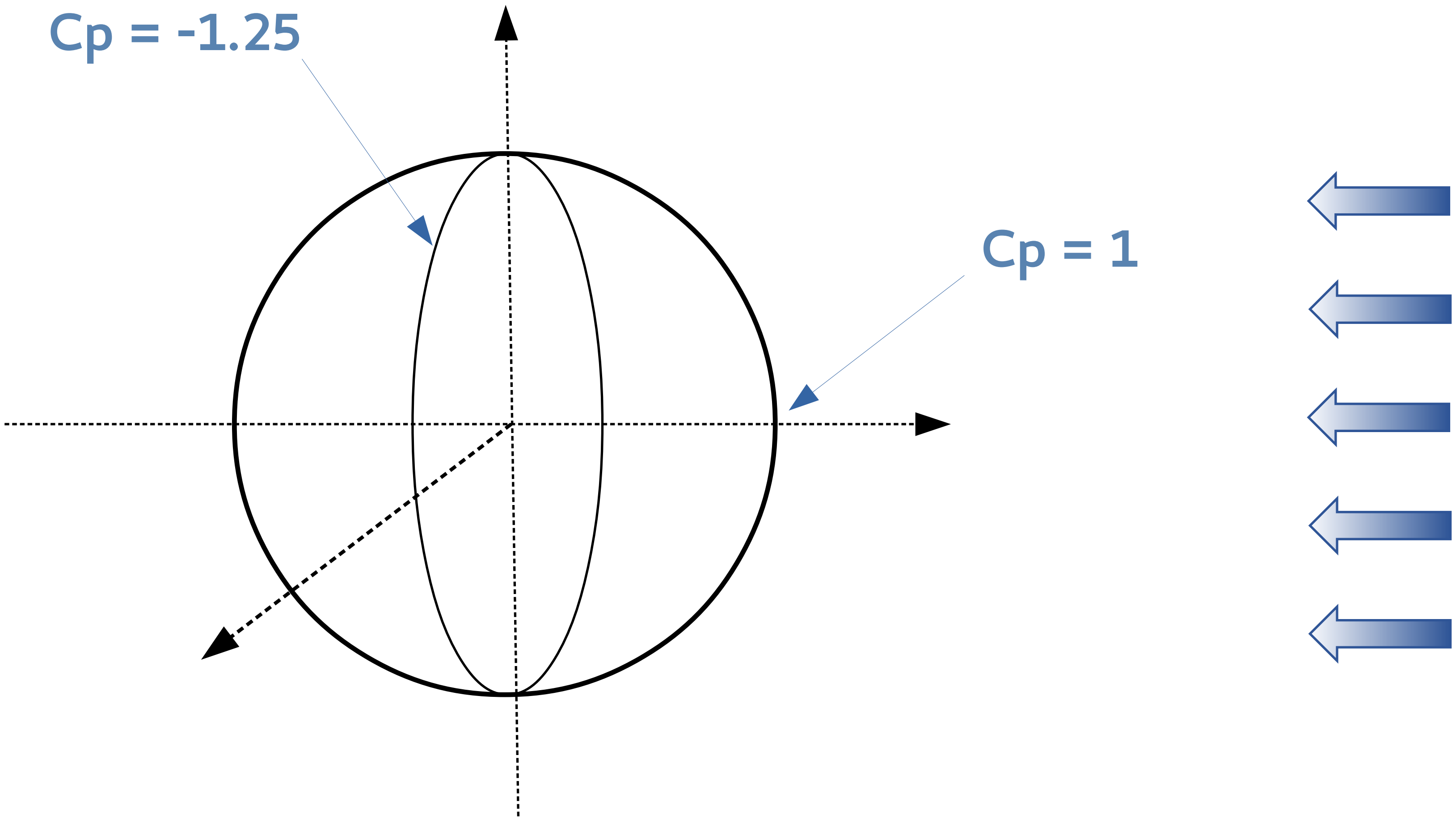
→ Wings can be modelled either as thick Dirichlet volumes or thin Neumann surfaces

6. Higher order methods are usually recommended over low order methods

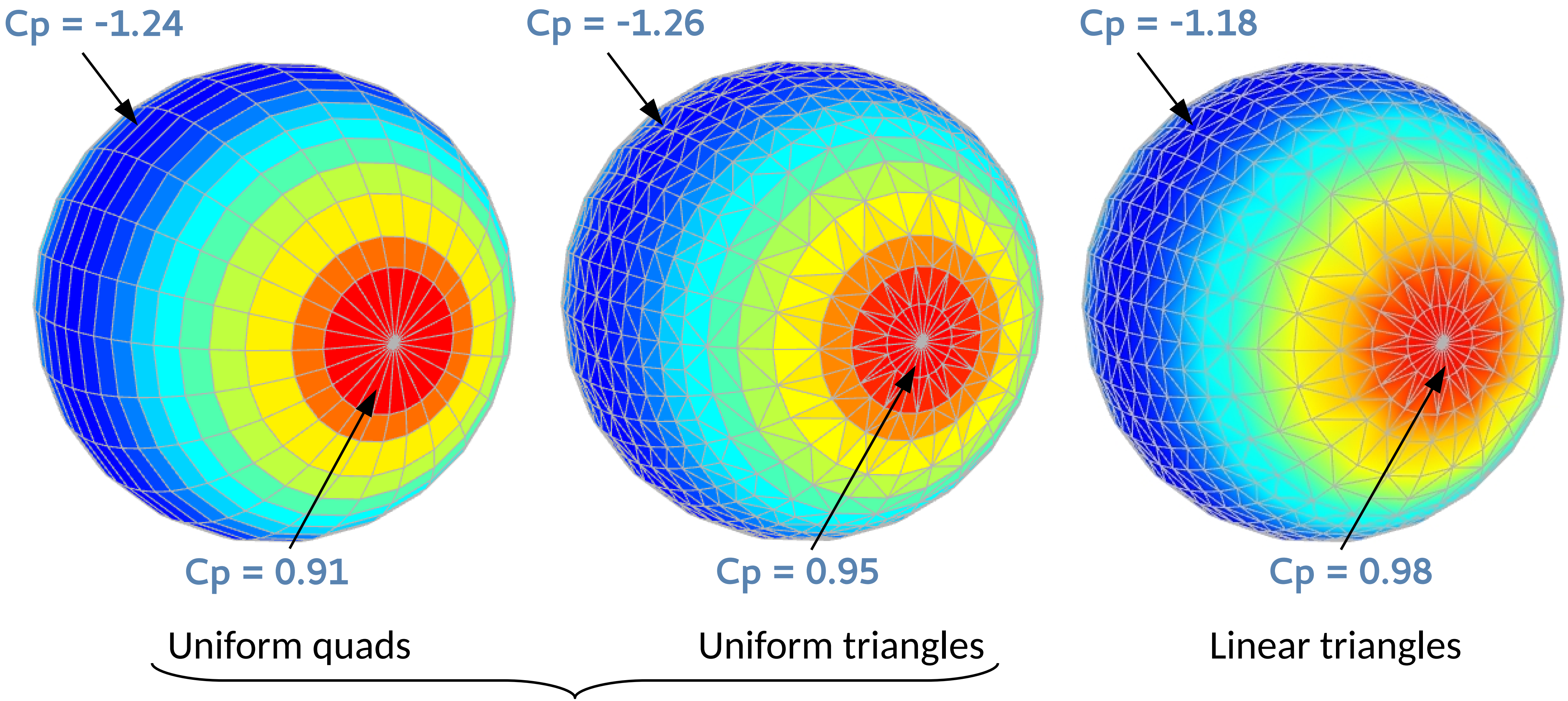
→ Prefer the uniform method for the preliminary design and the linear method in the final stages of the design

Pressure coefficients

Theoretical solution for the sphere in a potential flow



Pressure coefficients – « iso quad-mesh »

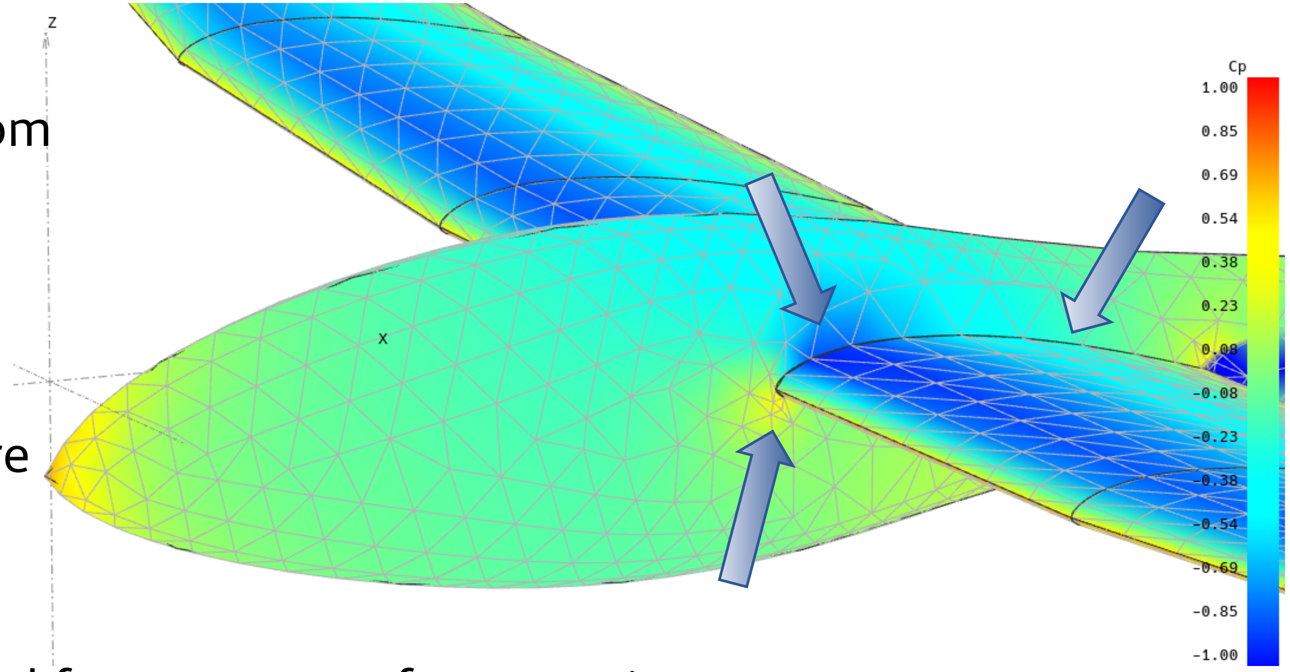


Not as precise in the area of high gradient

Potential for improvement

Pressure coefficients - Cp

- The pressure coefficients (Cp) are calculated from the surface gradient of the doublet densities.
- This requires that the elements be connected at their nodes
- The calculation is tricky when adjacent panels are not in the same plane such as at the junction of wings and fuselages



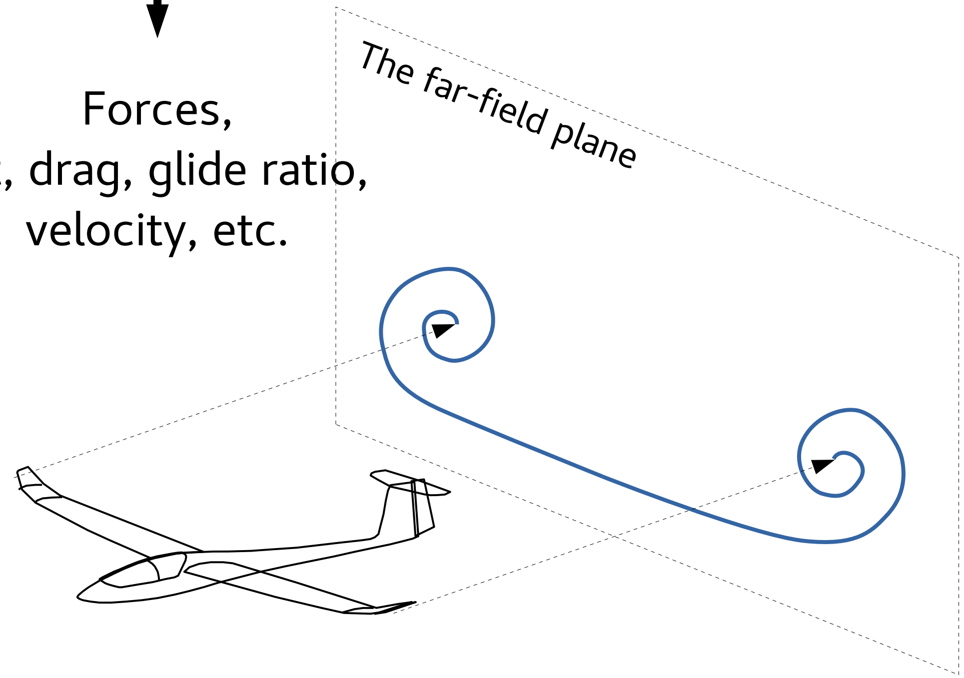
- It only impacts the moments which are calculated from pressure forces acting on the panels.
- It does not impact the lift and drag calculated in the far-field plane
- Potential to improve the precision of the Cp coefficients in the linear case
  → to be evaluated in the β-phase

Doublet densities

A panel analysis solves the linear system for the doublet densities on the panels. All other results are deduced from the doublet densities.

"Off-body" results calculated downstream in the far-field plane

Forces,
lift, drag, glide ratio,
velocity, etc.



"On-body" results calculated on the surfaces

Surface gradient of the doublet density

Pressure coefficients,
panel forces, moments